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Understanding Inflation Convexity

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Introduction

Inflation derivatives are products that derive their values from inflation indices, e.g., CPURNSA (or simply, CPI). Increasingly common, these products help investors retain the purchasing power of their currency and/or profit from their views on inflation. Like interest rate derivatives, inflation derivatives may among others make payments based on CPI readings at specified times; periodically exchange cash flows in year-over-year swaps based on inflation returns over the calculation periods; offer inflation protection in call or put options that pay off when the inflation rates reach their strike levels; or provide high yields in structured inflation notes.

Many inflation products involve payoffs that depend on the level of the inflation return, in particular, the year-over-year change of CPI. Because CPI curves are constructed from standard market instruments that price the levels of final CPI readings, the year-over-year CPI rates determined on a CPI level curve have a convexity premium. The magnitude of the convexity, which can be significant to pricing inflation derivatives, needs be explored in a consistent framework – thus the focus of this paper.

There have been few published works on inflation modeling. Jarrow and Yildirim (2003) theorize in an HJM framework that the inflation rate is an exchange rate between the real economy and the nominal economy. Separately, Belgrade, Benhamou and Koehler (2004) propose a multi-asset approach to price inflation derivatives. The JY model is appealing in that treating the inflation index as an exchange rate is intuitive, and is a natural extension of arbitrage-free interest rate and foreign exchange rate modeling for which research literature and data are abundant. However, because the real economy therein has to be inferred from the inflation market along with the nominal economy, hedging inflation derivatives in their approach becomes complicated. To address the disadvantages of the JY approach, Belgrade et al attempt a multi-asset approach in which consumer price level is treated as an index, much like one in equity modeling. As a result, complex correlation structures are introduced and hedging inflation derivatives that often pay off on inflation rates can be unintuitive and unstable. In a more recent article, Mercurio and Nicola Moreni (2006) propose a stochastic volatility model for forward consumer price indices with the volatility dynamics following a square-root process and derive closed-form formulae for inflation-indexed caplets. While their model can be used to extract convexity premium implied in the inflation option market, unfortunately, the inflation option market is young and much of the option implied volatility reflects the premium for illiquidity rather than market's true expectations of future inflation.

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In this article, I present an arbitrage-free methodology to quantify the convexity adjustment in inflation derivatives by working with inflation rates directly. Compared to the models proposed by other authors, my approach embodies a parsimonious correlation structure that is linked to market observables. My approach also makes it easy to employ in inflation rate products models related to existing interest rate-related models, a practicality of great benefit to inflation trading desks.

In sections below, I start with some definitions and present the derivations. I also use data for the USD market to estimate the size of convexity adjustments and analyze the relationship to other market variables.

CPI Level and CPI Rate

For most markets, zero-coupon inflation swaps are the most liquid instruments in the inflation market. A zero-coupon inflation swap allows swap counterparties exchange fixed payments for floating, inflation index-based payments. In a 5-year zero coupon swap, for example, party A may make payment to party B in 5 years in an amount proportional to the ratio of the final CPI level and today's CPI reading, while agreeing to receive a fixed payment from party B in 5 years. Because they reflect the market's expectations of future CPI levels, zero coupon swaps are the instruments utilized to construct CPI index curves.

I use $I(t,T)$ to denote the forward CPI level at observation time t for the settlement time T. For my purposes, I neglect the distinction between raw CPI and reference CPI levels. [1] Following the year-over-year swap conventions, I define the *forward* CPI rate for the period between U and T as:

$$R(t,U) \equiv \left[\frac{I(t,T)}{I(t,U)} - 1\right]\frac{1}{\Delta} \qquad [1b]$$

Where Δ is the day count fraction between horizons U and T. The fixing dates for the raw indices at U and T in general are not U and T due to CPI release lags. But in my framework, I assume the fixing dates are U and T without loss of generality. Because U < T and $I(T,U) \equiv I(U,U)$, the settlement rate for the given period is not fixed until the latter index is fixed. At T, the inflation rate for settlement is:

$$R(T,U) \equiv \left[\frac{I(T,T)}{I(U,U)} - 1\right]\frac{1}{\Delta} \qquad [1a]$$

[1] Most inflation products in the USD market use Reference CPI, which may be lagged and/or interpolated from the raw CPI levels. For example, the Reference CPI for USD for April 15 2006 is the linear interpolation of two raw CPI levels: those for the months of January 2006 and February 2006. The distinction between raw and reference CPIs is meaningful to trade settlement but is otherwise inconsequential to our mathematical derivations.

The definitions given by equations [1a-b] are widely used in inflation derivatives. In a year-over-year swap, for example, the recipient of the inflation rate payment is to receive payments for the period [U, T], at T that amounts to

$$N \cdot R(T,U) \cdot \Delta ,$$

where N is the notional amount for the transaction. In a call option, the holder of the option at the payment time is entitled to receive

$$N \cdot (R(T,U) - K)^{+} \cdot \Delta ,$$

from the option seller. More complex payoffs can be seen in exotic derivatives involving the inflation rates. In all cases the question arises that for a given period what the expected inflation rate is in relationship to the simple forward rate (cf. equation [1a]). Many market participants choose to neglect the differences between the simple forward rate and the expected inflation rate. As this paper will show, the difference known as convexity adjustment, while subject to market parameters, can be substantial.

Convexity Adjustment

In the T-forward measure, the co-period forward LIBOR rate $L(t,U)$ is a martingale:

$$\frac{dL(t,U)}{L(t,U)} = \sigma_L(t) \cdot dW_{Q^T} \qquad [2]$$

Where $\sigma_L(t)$ is a vector volatility in a multi-dimension space. The inflation rate convexity adjustment is just the difference between the expected CPI rate for the period and the naïve CPI rate as seen today,

$$\varphi(t,U) = E_{Q^T}\left[R(T,U) \middle| F_t\right] - R(t,U) \qquad [3]$$

On the other hand, the forward CPI index level $I(t,T)$ is the market's expectation at time t for horizon T. In the same T-forward measure $I(t,T)$ is a tradable instrument via zero-coupon swaps, and therefore a T-martingale. Likewise, in the U-forward measure $I(t,U)$ is a U-martingale. Assume $I(t,U)$ follows a geometric Brownian motion in the U-forward measure[2],

[2] Impact of volatility skew will be discussed later sections.

$$\frac{dI(t,U)}{I(t,U)} = \sigma_U(t) \cdot dW_{Q^U} \,, \qquad [4]$$

Where σ_U again is a vector and W_{Q^U} a multi-dimensional Brownian motion. In general, $I(t,U)$ is not a martingale in the T-forward measure.

Unlike other interest rates that represent funding costs, $R(t,U)$ in equation [1a] is the rate of change for a CPI index. For a given period this rate of change may be negative or positive. In other words, in a given period one may see price inflation or deflation.[3] With these considerations, I specify that $R(t,U)$ follows a normal stochastic dynamics:

$$dR(t,U) = \mu(t)dt + \zeta_R(t) \cdot dW_{Q^T} \qquad [5]$$

where $\zeta_R(t)$ is the volatility of $R(t,U)$. Clearly, $R(t,U)$ is not a martingale in the T-forward measure and the expectation of this rate will deviate from its simple-forward value at time t.

Now, via equation [1a],

$$I(t,T) = (1 + \Delta R(t,U))I(t,U)$$

It follows using the standard Ito calculus that

$$\frac{dI(t,T)}{I(t,T)} = \left[\sigma_U(t) + \frac{\Delta}{1+\Delta R(t,U)}\zeta_R(t)\right] \cdot dW_{Q^T} + \left[\frac{(\mu(t) + \sigma_U(t) \cdot \zeta_R(t))}{1+\Delta R(t,U)} - \frac{\sigma_U(t) \cdot \sigma_L(t) L(t,U)}{1+\Delta L(t,U)}\right]\Delta dt$$

,
[6]

where a change of measure between U- and T- forward measures has taken place:

$$dW_{Q^T} = dW_{Q^U} + \frac{\sigma_L(t)\Delta L(t,U)}{1+\Delta L(t,U)}dt$$

For $I(t,T)$ to be a martingale in the T-forward measure, the drift term in equation [6] must be strictly zero. It follows that

$$\mu(t) = \frac{\sigma_U(t) \cdot \sigma_L(t) L(t,U)}{1+\Delta L(t,U)}(1 + \Delta R(t,U)) - \sigma_U(t) \cdot \zeta_R(t) \qquad [7]$$

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[3] While one may argue that consumer prices don't decline over an extended period of time in a fiat currency system, for my purposes the inflation rates are year-over-year rates. In these yearly periods consumer prices can rise or decline.

As a result, the convexity adjustment can be explicitly written as

$$\varphi(t,U) \equiv E_{Q^T}\left[R(T,U)\middle|F_t\right] - R(t,U) = \int_t^U \mu(s)ds \qquad [8]$$

Note that the integration is to be carried over the horizon until U, the time at which the front CPI index fixes for the inflation rate and at which time the back CPI index automatically becomes a martingale.

Volatility of the CPI index

Evidently from equation [6], the volatility of $I(t,T)$ has a recursive relationship that can be written as

$$\sigma_T(t) = \left[\sigma_U(t) + \frac{\Delta}{1+\Delta R(t,U)}\zeta_R(t)\right] \qquad [9]$$

Denoting $\sigma_0(t)$ as the spot CPI volatility, I expand the equation for horizon U:

$$\sigma_U(t) = \sigma_0(t) + \sum_K^{U-1}\frac{\Delta}{1+\Delta R(t,K)}\zeta_R(t,K) \qquad [10]$$

Equation [10] relates the CPI index volatility to the inflation rate volatility, a relationship that can be quite useful when CPI level-based options must be priced. Keep in mind that these volatilities are instantaneous in a multi-dimensional space.

Plugging equation [10] into the equation [7], the unit-time convexity correction formula can now be expressed as a function of inflation rate volatility and the spot CPI index volatility $\sigma_0(t)$:

$$\mu(t) = \left[\sigma_0(t) + \sum_K^{U-1}\frac{\Delta}{1+\Delta R(t,K)}\zeta_R(t,K)\right]\cdot\left[\frac{\sigma_L L}{1+\Delta L}(1+\Delta R(t,U)) - \zeta_R(t,U)\right] \qquad [11]$$

With the dot products in equation [11], it becomes clear that the convexity adjustment is a function of the correlation structure between the spanning inflation rates and the LIBOR process. The overall convexity adjustment in theory can be positive or negative with the sign and the magnitude determined by the covariance terms in the equation. I shall elaborate these points later.

A Three-Asset Model

My derivations have been generic so far. I have assumed no knowledge about the correlation structures. Via arbitrage-free arguments, I have arrived at equations [7], [8], [10] and [11]. These equations are the core equations of my framework for convexity. If the volatility inputs and the correlation structures are known, the convexity adjustment can be directly calculated using these equations.

To start, I now make further insights into the convexity adjustment with some realistic assumptions. For my purposes, I assume that both the interest rate and the inflation rate follow their own one-factor dynamics. The two one-factor processes are correlated with each other, but within each process, all forwards are perfectly correlated. The one-factor dynamics should not be limiting because, after all, convexity adjustments have to do with all rates moving together. Steepening and twisting of the forward rate curves will have negligible impact on the magnitude of the convexity adjustment. Moreover, I assume that $\rho_{IL}(t)$ is the instantaneous correlation between the spot CPI index and the LIBOR rates; $\rho_{RL}(t)$ the instantaneous correlation between the spot inflation rate and the LIBOR rates; and $\rho_{IR}(t)$ the instantaneous correlation between the spot CPI index and the spot inflation rate. Note that these correlation coefficients are scalars directly related to market observables. As a result, equation [11] now is reduced to

$$\varphi(U) = \int_t^U \mu(s)ds = \int_t^U (A(s) - B(s))ds$$

$$A = \rho_{IL}(t)\frac{\sigma_0(t)\sigma_L(t)L(t,U)}{1+\Delta L(t,U)}(1+\Delta R(t,U)) + \rho_{RL}(t)\frac{\sigma_L(t)\Delta L(t,U)}{1+\Delta L(t,U)}\sum_K^{U-1}\frac{1+\Delta R(t,U)}{1+\Delta R(t,K)}\zeta_R(t,K) \quad [16]$$

$$B = \rho_{IR}(t)\sigma_0(t)\zeta_R(t,U) + \sum_K^{U-1}\frac{\zeta_R(t,K)\zeta_R(t,U)\Delta}{1+\Delta R(t,K)}$$

Equation [12] calculates convexity adjustment explicitly in terms of correlations, interest rate and inflation rate volatilities. Integration of the inflation rate and interest rate volatilities is conducted over the respective fixing horizons only.

Interestingly, with positive correlation scalars, equation [12] says that convexity is the result of two adjustments pulling in opposite directions. On one hand, the inflation rate's covariance with LIBOR adjusts the forward inflation rate ***higher***. This resembles the situation with LIBOR-in-arrears swaps. A change of measure from the U-horizon to the T-horizon reduces the expected CPI index for U when correlations are positive, and hence increases the expected inflation rate. On the other hand, the covariance among the inflation rates themselves adjusts the forward inflation rate ***lower***. When inflation rates rise, the purchasing power of the payments decreases, reducing the expected payout rate in today's dollar.

A Two-Asset Model

Moreover, the three correlation parameters in my framework are not equally important. Indeed, both ρ_{IL} and ρ_{IR} operate on a very short time scale. This means, practically, that they have negligible contributions to the size of convexity. On the other hand, the correlation between two rates, ρ_{RL}, plays an important role in both the direction and the size of the convexity adjustment. With these considerations, equation [16] can be further simplified to form a two-asset model:

$$
\begin{aligned}
\varphi(U) &= \int_t^U \mu(s)ds = \int_t^U (A(s) - B(s))ds \\
A &\approx \rho_{RL}(t)\frac{\sigma_L(t)\Delta L(t,U)}{1+\Delta L(t,U)}\sum_K^{U-1}\frac{1+\Delta R(t,U)}{1+\Delta R(t,K)}\zeta_R(t,K) \\
B &\approx \sum_K^{U-1}\frac{\zeta_R(t,K)\zeta_R(t,U)\Delta}{1+\Delta R(t,K)}
\end{aligned}
\quad [17]
$$

Equation [17] is quite amenable in that correlation-wise it just needs the correlation between the libor rate and the inflation rate. The correlation can be calibrated in the market, or estimated from historical data.

Keep in mind that in arriving at the formulations, I have assumed that the LIBOR rate follows a lognormal process and the inflation rate follows a normal process. The volatility and the correlation parameters are not functions of their respective rate levels. In a general situation, however, volatility skew is observed for both interest rates and inflation rates. Nonetheless, as evident from equation [17], the impact due to volatility skewing in LIBOR and inflation rate can be accounted for by incorporating into the instantaneous volatilities dependencies on the rate levels. It is, however, unclear how the covariance between LIBOR and inflation rate would behave in a volatility skewing environment. Further research and better market depth are needed before the effect of volatility skewing can be quantified.

While year-over-year rates are most common in inflation derivatives, shorter rates can be treated in my framework in a similar fashion. Nonetheless seasonal factors play an important role for shorter rates and uncertainty in these factors in general dominates the risks due to convexity adjustment.

How Significant Is Convexity

Two important parameters in this approach are volatility for the inflation rate and the correlation between LIBOR and the inflation rate. Unfortunately in today's market, there are no liquid instruments that price these two parameters. Call and put options on inflation rates are available and quoted in the market. But because their implied volatility reflects compensation for illiquidity, these options cannot be used as market expectations for the volatility of forward CPI rates.

I use two approaches to quantify convexity, historical and blended approaches. In the historical approach, I use estimates from historical data. In the blended approach, I use historical data and

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additional premium to account for current market expectations. In both estimates, I have to infer the volatility for forward CPI rates from that of the spot CPI rate in order to calculate convexity adjustment.

Over the years, economists have extensively studied inflation dynamics, e.g., whether inflation has a temporary or a sustained impact on nominal interest rates and economy at large, with particular interest paid to the issue of whether inflation rates can be characterized as a unit-root or a mean-reverting process. [4] While it was believed in earlier works by economists that inflation rate process cannot be rejected as a unit root process (cf., Nelson and Plosser (1982)), later studies (e.g., Culver and Papell (1997), Lee and Wu (2001)) establish that inflation rates do follow a mean-reverting process. Inflation rate deviations from the market perceptions of the central banks' target inflation rates dissipate within a finite time. In a more recent study, Dosshe and Everaert (2005) demonstrate that the inflation rate process mean reverts within a time span that ranges from 1 to 16 quarters for the US economy.

With these research results I assume that the spot inflation rate follows an Ornstein-Uhlenbeck process. Derivations to link between the desired volatilities are in standard texts and are not repeated here. I then estimate the spot volatility from the historical CPI data, and subsequently approximate the forward CPI volatility through a mean-reversion coefficient and the spot volatility.

Figure 1 shows the 24-month moving spot volatility for the spot 1Y inflation rate and 24-month moving correlation between spot 6M libor rates and the 1Y CPI rate, calculated from the NSA-CPI data from 1971 to 2000 for the US (data sources: US Bureau of Labor Statistics and the Federal Reserve). Historically, the correlation between LIBOR and inflation rate is quite low. The correlation rose to near 50% in the mid 1970s, only to drop back to near zero when the Fed raised rates towards late 70s. In the early 80s, this correlation dipped below zero and stayed below zero for a number of years. For more recent years this correlation on average has been near 20%. Nonetheless, the graph depicts a sensible economic picture in which inflation picks up and LIBOR rates rise in "good" expansion years, and price increases slow with interest rate cuts in lean times. Based on the more recent experience, I assume the correlation at 20% in my analysis.

[4] A unit-root process would propagate short-term shocks to the infinite horizon, whereas a mean-reverting process diffuses the short-term shocks within a finite time.

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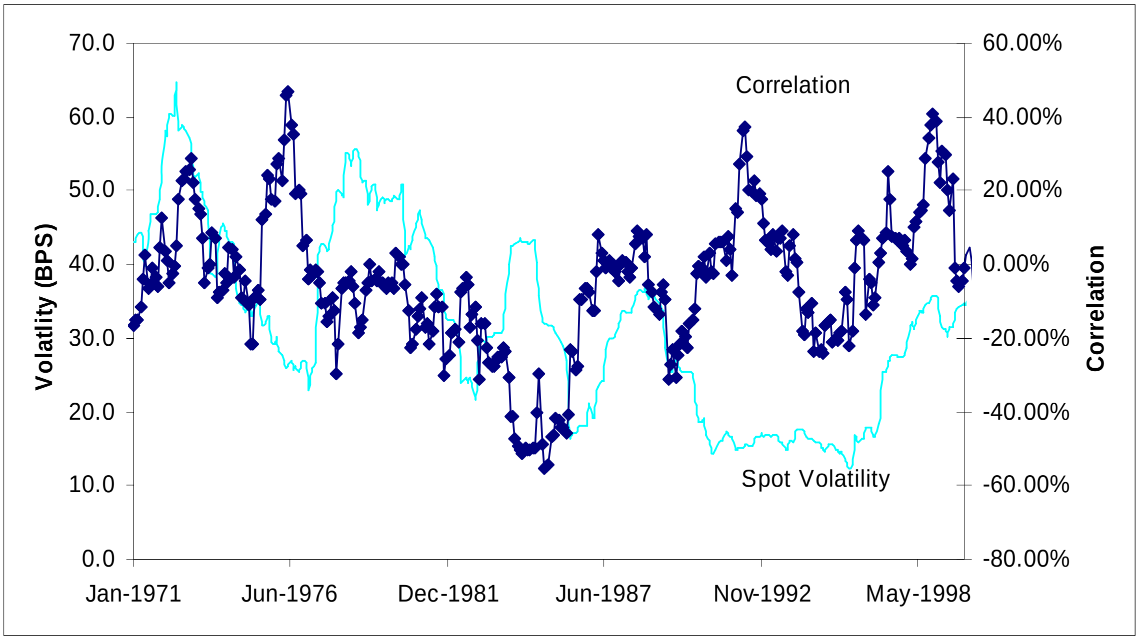

Figure 1: historical 24-month rolling 1Y inflation rate volatility (bps, left axis) and historical 24-month rolling correlation (right axis) between LIBOR and inflation rate.

Over the 30 years, interestingly enough, the volatility for the 1Y inflation rate generally follows the trend in the correlation level: when the correlation level trends higher, the volatility rises; when the correlation level slumps, the volatility also drops. Nonetheless, the volatility has been largely confined between 20 bps and 60 bps. More recently this number has been on average near 50 bps.

To estimate convexity adjustment, I use the implied volatility for LIBOR and the current market data to construct the CPI curve. For the inflation rates, as I discussed, I adopt the historical and the blended approaches to quantify the volatility for inflation rates. In the historical approach, I set the inflation rate volatility at 50 basis points and use a mean reversion parameter at 25% to calculate the volatility for the forward inflation rates. In the blended approach, I use a more recent historical volatility for the 1Y inflation rate (also at 50 bps), and add a market premium of 30 bps to account for the current market expectations. The total 80 bps for the spot inflation rate volatility is comparable to the current market expectation (at the time of this writing) for the volatility of LIBOR rates.

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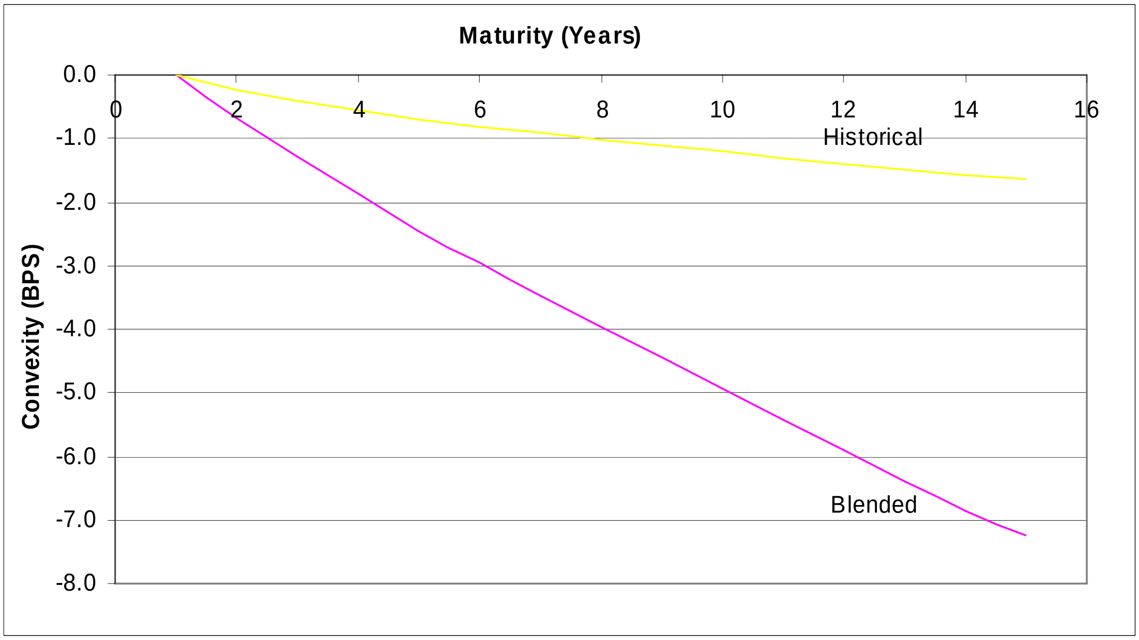

Figure 2: Convexity adjustments in bps for 1Y inflation rate as a function of the rate maturity in years in two different approaches: blended and historical approaches.

Plotted for the US market in figure 2 is the convexity adjustment in basis points as a function of the settlement maturity in years for 1Y inflation rates under the two approaches. The yellow line represents the adjustment in the historical approach and the purple line illustrates the result in the blended approach. Both approaches use the same inflation curve built with zero-coupon swap rates (data courtesy of IXIS FP), roughly between 3.1% and 3.3% at the time of this writing.

Evidently, as the graph shows, convexity adjustment at the assumed correlation level 20% increases in size as a function of the rate maturity and is always negative. Recall that the convexity adjustment is the difference between the expected inflation rate and the naïve forward inflation rate. Increasingly negative convexity adjustment is the result of the fact that the index bracketing the year-over-year rate is an accumulative effect of rates in prior years. Therefore, ***inflation receivers get higher nominal payments with higher inflation rates, but these payments will be worth less, again due to higher inflation rates.***

The sizes of the convexity adjustment in two approaches are quite different. For shorter maturities, the adjustments are only a few basis points with the historical approach being lower. The blended approach diverges away from the historical approach with long maturities. I attribute the divergence between the two approaches as due to differences in volatility levels adopted for each approach. For the blended approach, the covariance among the spanning inflation rates for a given maturity become more dominant, making the convexity adjustment more negative.

Convexity adjustment may also be inferred using option-implied volatility for the inflation rates. As I have pointed out before, unlike interest rate options, inflation options are at an early stage and are quite illiquid. While inflation cap floors are available from inter-bank broker quotes, the implied volatilities for inflation options build in a significant liquidity premium. Therefore the sizes of convexity adjustment under the approach using option-implied volatility parameters will likely be substantially higher than those in the blended and the historical approaches. It is difficult to ascertain to what degree the parameters reflect the true market expectations, rather than premium due to illiquidity and/or lack of two-way flows. For these reasons I believe that the true convexity adjustment would be close to the blended approach if dynamically hedging inflation becomes feasible.

Finally, I document below the impact of the correlation (between LIBOR and the inflation rate) to the size of convexity adjustment. For intermediate (5Y) and long (10Y) maturities, in my blended approach the 1Y inflation rate has a negative convexity at -2.4 and -4.9 bps respectively. When the correlation shifts lower, the convexity adjustment increases in size (more negative). The reverse is true when the correlation becomes higher.

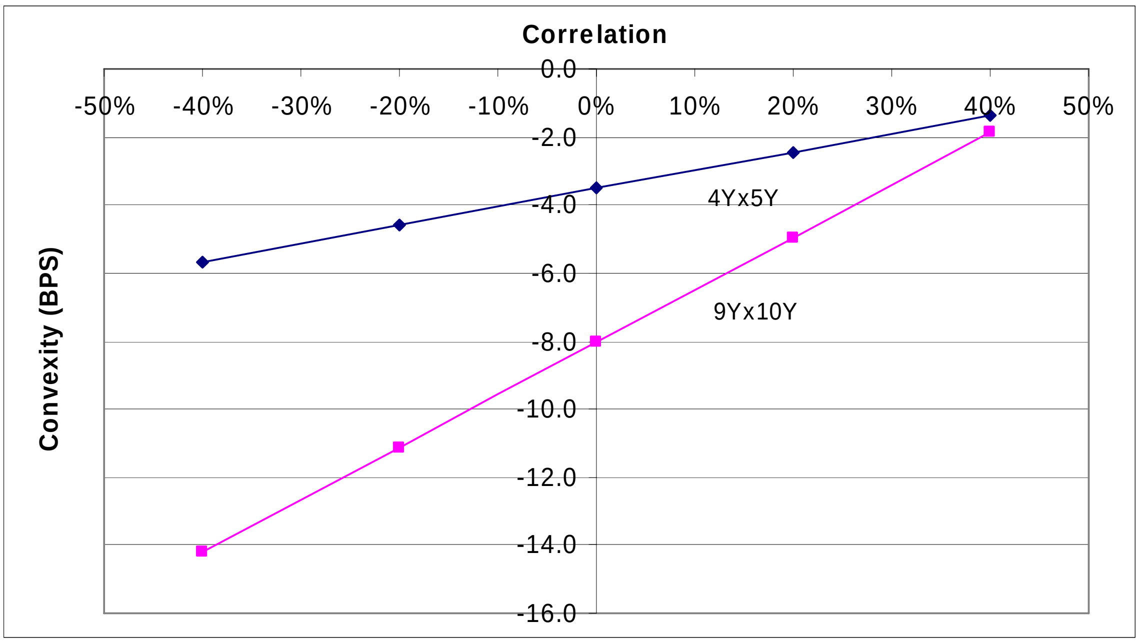

Figure 3: Convexity adjustment as a function of correlation between LIBOR and inflation rate. Two lines are the adjustments for 1Y inflation rate maturing in 5Y and 10Y respectively.

Recall that in figure 1 I demonstrate that historically, the correlation between LIBOR and the inflation rate has been small or negative in lean times. This graph suggests that with everything being equal, the market should be content to receive a lower rate in exchange for paying inflation in lean times, a conclusion consistent with intuitions. Granted, in lean times the implied volatility for inflation rates may drop as inflation products would be expected to have inferior returns, this effect would help stabilize the inflation convexity. Currently there is no active market for this correlation coefficient. Although inter-bank year-over-year vs. zero-coupon switchers are quoted and could come to play this role, the market lacks depth at the time of this writing.

Conclusions

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I outline a methodology to calculate the convexity adjustment to the inflation forward rates. Our methodology is simple and easy to implement, and can be integrated with existing interest rate models to price inflation products. The size of the adjustments is a function of inflation rate volatility and its correlation with LIBOR. Using historical data, it appears that the average adjustment for 10Y year-over-year deal should be about -1 basis point per year. My blended approach, which combines the historical with current market parameters, suggests a bigger convexity adjustment, at roughly -2.5 basis points per year for a 10Y swap deal. Nonetheless, the magnitude and the direction of adjustment are sensitive to volatility assumptions. In general, the net result of several contributing factors in convexity is to adjust the naïve forward rates *lower*. Inflation options at the moment lack the depth and are not useful in inferring market's expectations of inflation convexity adjustment.

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